1. The illustration to the right shows a spherical hollow inside a lead sphere of radius \( R = 4.00 \) cm. The mass of the sphere before hollowing it out was \( M = 2.95 \) kg. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass \( m = 0.431 \) kg that lies a distance \( d = 9.00 \) cm from the center of the lead sphere?

2. Certain neutron stars are believed to be rotating at about 1 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during its rotation?

3. A hypothetical triple-star system consists of two stars, each of mass \( m \), revolving in the same circular orbit of radius \( R \) around a central star of mass \( M \). The two orbiting stars are always at opposite ends of a diameter of the orbit. Derive an expression for the period of revolution of the stars.

4. Three identical particles of mass \( M \) form an equilateral triangle that rotates around the triangle’s center as the particles move in a common circle about the center. The triangle has an edge length \( L \). What is the speed of the particles?

5. Consider a homogeneous thin ring of material with a total mass \( M \) and radius \( R \), as shown to the right. What gravitational attraction does that ring exert on a particle of mass \( m \) on the ring’s central axis a distance \( x \) from the ring’s center?
6. One way to attack a satellite in orbit is to launch a cloud of pellets in the same orbit as the satellite, but in the opposite direction. Suppose a satellite in circular orbit 500 km above the Earth’s surface collides with a pellet having a mass of 4.0 g. What is the kinetic energy of the pellet in the reference frame of the satellite just before collision? Compare this energy to that of a 4.0 g bullet fired from a rifle with a muzzle velocity of 950 m/s.

7. Three point particles are fixed in x-y space. Particle A has mass \( m \), B has mass \( 2m \), and C has mass \( 3m \). A new particle D with mass \( 4m \) is to be placed such that the net gravitational force on particle A is zero. At what x and y coordinates should particle D be placed?

8. Mass \( M \) is distributed uniformly throughout a thin rod of length \( 2L \). A particle with mass \( m \) is at a point that is a distance \( a \) from the vertical centerline of the rod along the rod’s perpendicular bisector. Calculate the gravitational force that the particle experiences due to the mass of the rod.
Before hollowing out
Sphere \( M_0 = 2.95 \text{ kg} \)
\( m = 0.431 \text{ kg} \)

Spherical hollow inside a solid sphere of \( R = 4.00 \text{ cm} \)
cavity has radius \( \frac{R}{4} \)

\[
V = \frac{4}{3} \pi R^3 \quad \frac{M_0}{(4.00 \text{ cm})^3} = 0.0461 \text{ kg} \quad \text{density} \quad \frac{0.369}{0.0461 \text{ kg}}
\]

Volume \( V_{	ext{hollow}} = 0.0461 \text{ kg} \cdot (2.00 \text{ cm})^3 = 0.000045 \text{ kg} \text{ cm}^3 \) for hollow part.

For the hollow part,

\[
F_5 = \frac{G M m}{r^2} = \frac{(6.67 \times 10^{-11}) (2.95 \text{ kg}) (0.431 \text{ kg})}{(0.09 \text{ cm})^2}
\]

\[
F_5 = 1.047 \times 10^{-8} \text{ N}
\]

For the hollowed part

\[
F_H = \frac{(6.67 \times 10^{-11}) (0.431 \text{ kg}) (0.369 \text{ kg})}{(0.07 \text{ cm})^2}
\]

\[
F_H = 2.165 \times 10^{-9} \text{ N}
\]

\[
F = F_5 - F_H = 8.305 \times 10^{-9} \text{ N}
\]
2. If \( g \to 0 \), still flies if:

\[
\omega = 2\pi \text{ rad/s.}
\]

\[
\frac{mv^2}{r} \to mrw^2
\]

\[
mrw^2 = \frac{GMm}{r^2} = 0
\]

\[
Mrw^2 = \frac{GMm}{r^2} \Rightarrow M = \frac{r^3w^2}{G}
\]

\[
M = \frac{\left(2.0 \times 10^3 m\right)^3 (2\pi)^2}{6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}}
\]

\[
M = 4.73 \times 10^{24} \text{ kg}
\]

3. For this star:

\[
F = G\frac{mm}{R^2} + G\frac{m^2}{(2R)^2}
\]

\[
\frac{mv^2}{R} = F = G\frac{m}{R^2} \left(M + \frac{m}{4}\right)
\]

\[
V = \frac{2\pi R}{t}
\]

\[
\frac{4\pi^2 R^2}{t^2} = G\frac{m}{R^2} \left(M + \frac{m}{4}\right)
\]

\[
\frac{4\pi^2 R}{G\left(M + \frac{m}{4}\right)} = T^2
\]

\[
\sqrt{\frac{4\pi^2 R^3}{G\left(M + \frac{m}{4}\right)}} = T \quad \text{or} \quad \frac{2\pi R^{3/2}}{\sqrt{G\left(M + \frac{m}{4}\right)}}
\]
\[ R = \frac{L}{2 \cos \theta} = \frac{L}{2 \frac{V_0^2}{2}} = \frac{L}{\sqrt{3}} \]

\[ F = \frac{G M^2}{L^2} \]

\[ F_{net} = \frac{2 G m^2 \cos \theta}{L^2} \]

\[ F_{net} = \frac{2 G m^2 V_0^2}{L^2} \]

\[ F_{net} = \frac{6 G m^2 V_0^2}{L^2} \]

\[ \frac{M V^2}{L} = F_{net} = \frac{G M^2 V_0^2}{L^2} \]

\[ \frac{M V^2}{L^2} = \frac{G M^2 V_0^2}{L^2} \]

\[ \frac{V^2}{L} = \frac{G M}{L} \Rightarrow V = \sqrt{\frac{G M}{L}} \]

\[ \text{all mass is concentrated at}\]

\[ M \]

\[ x \]

\[ d = \sqrt{x^2 + y^2} \]

\[ F \]

\[ \frac{mg}{\sqrt{x^2 + y^2}} \]

\[ F \cos \theta = \frac{x}{\sqrt{x^2 + y^2}} \]

\[ F = \frac{G M m}{L^2} \frac{x}{\sqrt{x^2 + y^2}} = \frac{G M m x}{(x^2 + y^2)^{3/2}} \]
6. \( d = 6.37 \times 10^6 \text{m} + 0.5 \times 10^6 \text{m} \quad r_{\text{total}} = 6.37 \times 10^6 \text{m} \)
   \( d = 6.57 \times 10^6 \text{m} \).

\[
\frac{\mu v^2}{d^2} = \frac{6M_e m}{d^2} \quad \frac{v^2}{d} = \frac{6M_e}{d} \quad v = \sqrt{6M_e/d}
\]

relative to the satellite, \( v_s = 2V = 2\sqrt{6M_e/d} \)

\[
K_E = \frac{1}{2} m v_s^2 = \frac{1}{2} m \left(2\sqrt{6M_e/d}\right)^2 = 2m6M_e
\]

\[
K_E = 2 \left(0.004\text{ kg}\right)\left(5.98 \times 10^{24}\text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(6.57 \times 10^6 \text{m}\right)
\]

\[
K_E = 4.64 \times 10^5 \text{ J}
\]

\[
K_{\text{required}} = \frac{1}{2} m v^2 = \frac{1}{2} \left(0.004\text{ kg}\right)\left(350 \text{ m/s}\right)^2 = 1905 \text{ J}
\]

\[
\frac{4.64 \times 10^5}{1905} \approx 25.7 \times 1
\]

7. \( \begin{array}{c}
1 \\
2 \\
3 \, 1.5
\end{array} \)

\[
F_{y1,2} = \frac{G(1)(2)}{1} = 2G
\]

\[
F_{y1,3} = -\frac{G(1)(3)}{(1.5)^2} = 1.33 G
\]

\[
F_{\text{net}} = \sqrt{2^2 + 1.33^2} = 2.402, \quad \theta = \tan^{-1} \frac{2}{1.33} = 56.38^\circ
\]

First moment balance,

\[
2.402 \times 1 = \frac{(1)(4)}{d^2} \quad d = 1.29
\]

\[
\text{for}\ n = 1.07 \uparrow
\]

\[
+0.714 \uparrow
\]

\[
\text{for}\ \theta = 56.38^\circ\ \text{at} \quad 1.29
\]

\[
+0.714 \uparrow
\]
\[ \lambda = \frac{M}{2L} \]

\[ dx = \lambda dx \]

\[ r = \sqrt{x^2 + a^2} \]

\[ F_x = F \cos \theta \]

\[ dF_x = dF \frac{a}{r^2} \]

\[ df_x = 6 \frac{mdm}{r^2} a \]

\[ F_x = \int \frac{Gmdm}{X^2 + a^2} \frac{\lambda dx}{\sqrt{X^2 + a^2}} = \frac{GMa \lambda}{2L} \int_{-L}^{+L} \frac{dx}{(X^2 + a^2)^{3/2}} \]

\[ F_x = \frac{GMa}{2L} \int_{-L}^{+L} \frac{dx}{(X^2 + a^2)^{3/2}} \]

\[ F_x = \frac{6wM}{a \sqrt{a^2 + L^2}} \]

If \( a \gg L \), then \( F_x \approx \frac{6wM}{a^2} \)